Interpretation of Quasifree Proton-Deuteron Scattering*

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Proton-deuteron inelastic scattering is studied in impulse approximation, using the formalism of Cromer. The S-wave final-state interaction of the two target nucleons is included. Expressions are evaluated for a scattering energy of 142 MeV. The results for quasifree scattering are compared with calculations of Everett. Results for quasifree p-p scattering are compared with measurements of cross section, polarization, and triple-scattering parameters R and A. Agreement for all but cross section is good, that for cross section is qualitatively right. Results for quasifree p-n scattering are used to obtain corrections relating quasifree measurements of P, R, A, and D to the free n-p parameters. These corrections are applied to the existing measurements of P, R, and A. Frequently, the corrections are larger than the experimental errors, and must be made before the data can be interpreted as n-p data. Scattering in which the target nucleons are left in a relative singlet spin state is considered, polarization parameters are evaluated, and methods for measuring them discussed.

I. INTRODUCTION

IN the last few years, an extensive series of proton-deuteron scattering experiments has been performed by Wilson and his collaborators[1–6] at the Harvard Cyclotron Laboratory. This includes measurements of the elastic proton-deuteron differential cross section and polarization,[1] I_D and P_D, and triple scattering parameters,[2] R_D and A_D; measurements of the small-angle slightly inelastic cross section, $d\sigma/d\Omega dE$, of inelastically scattered protons near threshold[3]; and measurements of the "free" proton-proton cross section,[4] "free" proton-proton[4] and proton-neutron[5] polarization, and the triple-scattering parameters[6] R and A, obtained by quasifree proton-deuteron scattering. The two preceding papers[6,2] (hereafter referred to as I and II) are a part of this series. In this paper we shall discuss how these experiments, and especially the quasifree measurements, can be interpreted in terms of free nucleon-nucleon scattering parameters.

The impulse approximation can be used to relate proton-deuteron scattering to the free nucleon-nucleon scattering amplitudes. It can be shown,[7] for instance, that the elastic proton-deuteron measurements, at small angles in the laboratory system, are related to the free p-p and n-p scattering matrices M_{pp} and M_{np} by

$$I_D S_D(ij) = \Omega F^2(q) \Sigma_t S_t(ij), \tag{1.1}$$

where

$$\Sigma_t S_t(ij) = \tfrac{1}{6}\,\mathrm{Tr}[(M_{np}^\dagger + M_{pp}^\dagger)\Lambda_t \sigma_i (M_{np}+M_{pp})\Lambda_t \sigma_j]\,. \tag{1.2}$$

Here Ω is a kinematical factor defined in II, Table VIII, and $F(q)$ is the deuteron form factor at the momentum transfer q. Λ_t is the triplet projection operator for the spins of the target nucleons and σ_i is the ith component of the spin operator for the incident proton.

We let $\mathbf{P}_0$ and $\mathbf{P}_0'$ be the initial and final momenta of the incident proton in the nucleon-nucleon c.m. system and define $\sigma_0 = 1$, $\sigma_1 = \boldsymbol{\sigma}\cdot\mathbf{p}$, $\sigma_2 = \boldsymbol{\sigma}\cdot\mathbf{q}$, and $\sigma_3 = \boldsymbol{\sigma}\cdot\mathbf{n}$, where $\mathbf{p}$, $\mathbf{q}$, and $\mathbf{n}$ are unit vectors in the directions $\mathbf{P}_0+\mathbf{P}_0'$, $\mathbf{P}_0'-\mathbf{P}_0$, and $\mathbf{P}_0\times\mathbf{P}_0'$, respectively. Then we have $S_t(00) = 1$, $S_t(30) = P_D$, $S_t(33) = D_D$, and[8]

$$R_D = S_t(22)\cos\tfrac{1}{2}\theta + S_t(12)\sin\tfrac{1}{2}\theta\,, \tag{1.3}$$

$$A_D = -S_t(22)\sin\tfrac{1}{2}\theta + S_t(12)\cos\tfrac{1}{2}\theta\,, \tag{1.4}$$

where θ is the scattering angle in the nucleon-nucleon c.m. system and D, R, and A are the triple-scattering parameters defined by Wolfenstein.[9] If the angle between momentum transfer $\mathbf{q}$ and scattered particle direction differs from 90° (as occurs when θ becomes large), the expressions for R and A become more complicated, as explained in II. There, expressions for

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the various elastic-scattering parameters are listed, evaluated, and compared with experiment.

In a previous paper[10] (hereafter referred to as A), a theory of inelastic proton-deuteron scattering which includes the final S-state interaction of the resultant neutron-proton system has been used to analyze the slightly inelastic cross-section measurements of Stairs *et al.*[3] It was shown in A that such a theory gave a quantitative fit to the data and that the data could be interpreted as giving a measurement of the parameter

$$\tfrac{1}{3}\Sigma_s=\tfrac{1}{6}\,\mathrm{Tr}[(M_{np}^\dagger+M_{pp}^\dagger)\Lambda_s(M_{np}+M_{pp})\Lambda_t], \quad (1.5)$$

where Λ_s is the singlet projection operator for the spins of the target nucleons. In Sec. IV we shall discuss how measurements of slightly inelastic p-d scattering using polarized protons could be used to measure singlet parameters analogous to the triplet parameters of Eq. (1.2)

$$\tfrac{1}{3}\Sigma_s S_s(ij)=\tfrac{1}{6}\,\mathrm{Tr}[(M_{np}^\dagger+M_{pp}^\dagger)\Lambda_s\sigma_i(M_{np}+M_{pp})\Lambda_t\sigma_j]. \quad (1.6)$$

The quasifree scattering experiments attempt to measure "free" nucleon-nucleon parameters. Quasifree scattering is an inelastic process in which the incident particle interacts with one of the target nucleons essentially as if the latter were free. Two detectors are placed about 90° apart to detect the scattered particle and the recoil nucleon in coincidence. In the spectator-model approximation (SMA), the cross section for the incident particle to scatter from the target neutron into a solid angle $d\Omega_p$, while the neutron goes into the solid angle $d\Omega_n$ with energy E_n, is

$$d^3\sigma_{\mathrm{SM}}\equiv\frac{d^3\sigma}{d\Omega_n d\Omega_p dE_n}=KG(k_p')^2\sigma_{np}. \quad (1.7)$$

Here, K is a kinematical factor which will be defined later, $G(k_p')$ is the deuteron wave function in momentum representation, k_p' is the final momentum of the target proton (the spectator particle), and σ_{np} is the free neutron-proton differential cross section. The inelastic cross section is sharply peaked at values of E_n for which $k_p'=0$. This is the quasifree peak and is the region in which Eq. (1.7) is most valid. Similar considerations hold also for scattering from the target proton.

The polarized cross section is given by

$$d^3\sigma_{\mathrm{SM}}S_{\mathrm{SM}}(ij)=KG(k_p')^2\sigma_{np}S_{np}(ij), \quad (1.8)$$

so that $S_{\mathrm{SM}}=S_{np}$. This means that in the SMA the polarization parameter measured in a quasifree proton-deuteron experiment is equal exactly to the free parameter.

In order to establish the limits of validity of Eqs. (1.7) and (1.8), Kuckes *et al.*[4] measured the differential cross section $d^3\sigma/d\Omega_{p1}d\Omega_{p2}dE_{p2}$ for quasifree scattering from the target proton and compared the result to the predictions of the SMA. They found that the cross sections were about 15% smaller than those predicted by Eq. (1.7). Their quasifree measurements of p-p polarization, however, were in good agreement with the free p-p measurements.

Kuckes and Wilson[5] have used this method to obtain measurements of the p-n polarization, and Lefrançois *et al.*[6] have measured the p-n and p-p triple-scattering parameters R and A. In order to interpret these measurements properly, it is necessary to have a theory which goes beyond the SMA so that corrections to the quasifree values of these parameters can be calculated.

In Sec. II the inelastic proton-deuteron theory of A is used to derive a better expression for the quasifree cross section by taking account of the final S-state interaction of the target nucleons. In Sec. III the results are compared to similar calculations of Everett[11] and are found to be in general agreement at small scattering angles where the two theories are expected to give similar results. The theory is then used to calculate the quantities $\Delta S_{pp}=S_{pp}-S_{\mathrm{QF}}$, where S_{pp} is a free p-p parameter and S_{QF} is the value predicted by the theory for a quasifree experiment. These values are compared with experimental values of ΔS_{pp}, obtained from the difference between the values obtained in free and quasifree experiments. Finally, the quantities ΔS_{pn} are calculated and used to obtain the free n-p values of P, R, and A from the quasifree values.

II. THEORY OF QUASIFREE SCATTERING

In this section we derive the laboratory differential cross section $d^3\sigma/d\Omega_n d\Omega_p dE_n$ for the process in which an incident proton inelastically scatters from a deuteron through an angle θ_p into the solid angle $d\Omega_p$, while the target neutron recoils with an energy in the interval dE_n into the solid angle $d\Omega_n$. Let $\mathbf{P}$ and $\mathbf{P}'$ be the initial and final momenta of the incident proton and let $\mathbf{k}_p'$ and $\mathbf{k}_n'$ be the final momenta, respectively, of the target proton and neutron, all momenta being given in the laboratory system.

Then, from Eq. (2.17) of A, one obtains

$$d^3\sigma\equiv\frac{d^3\sigma}{d\Omega_p d\Omega_n dE_n}=\tfrac{1}{6}K\,\mathrm{Tr}(\mathfrak{M}^\dagger\mathfrak{M}), \quad (2.1)$$

where K is the kinematical factor[12]

$$\begin{aligned}K&=(P/mP_0^2)k_n'^2P'^2(\partial k_n'/\partial E_n')(\partial P'/\partial E_f)\\&=mPk_n'(P'/P_0)^2\chi[(1+\chi)P'-P\cos\theta_p\\&\qquad+k_n'\cos\theta_{\mathrm{inc}}]^{-1}. \quad (2.2)\end{aligned}$$

Here, E_f is the final energy of the entire system, θ_{inc} is the angle between the directions of the scattered proton

[10] A. Cromer, Phys. Rev. **129**, 1680 (1963), hereafter referred to as A.

[11] A. Everett, Phys. Rev. **126**, 831 (1962).

[12] Throughout this paper we use a system of units in which $\hbar=c=1$.

and the recoil neutron, and

$$\chi=[(k_p'^2+m^2)/(P'^2+m^2)]^{1/2}.$$

If we neglect both the final-state interaction between the two outgoing target particles and the scattering of the incident proton more than once, the scattering matrix $\mathfrak{M}$ has the simple form

$$\mathfrak{M}=[M_{np}(q)G(k_p')+M_{pp}(q)G(k_n')]\Lambda_t. \quad (2.3)$$

Here, $G(k)$ is the Fourier transform of the deuteron wave function, $M_{np}(q)$ and $M_{pp}(q)$ are the free center-of-mass neutron-proton and proton-proton scattering matrices evaluated for a momentum transfer $\mathbf{q}=\mathbf{P}-\mathbf{P}'$, and Λ_t is the triplet spin-projection operator for the spins of the target nucleons, since initially they are in a triplet spin state.

Actually M_{np} and M_{pp} are not the free-particle scattering matrices since the latter always involve initial and final states of the same energy, whereas this is not the case for M_{np} and M_{pp} in Eq. (2.3). It is shown in A that M_{np} and M_{pp} represent scattering between two states which differ in energy by $(\epsilon_B+k_p'^2/m)$ and $(\epsilon_B+k_n'^2/m)$, respectively. Here, ϵ_B is the absolute value of the deuteron binding energy. Thus, M_{np} will be approximately equal to the free-scattering matrix when $k_p'=0$. But, this is just the condition for quasifree scattering of the incident proton from the target neutron since $G(k_p')$ is a maximum for $k_p'=0$ and falls to zero rapidly for $k_p'>(m\epsilon_B)^{1/2}$. Thus, M_{np} is expected to differ from the free neutron-proton scattering matrix only in a region where its contribution to the cross section is relatively small. Similar considerations hold for M_{pp}.

Using this expression for $\mathfrak{M}$ in Eq. (2.1), we get what we term the "Born approximation" cross section,

$$\begin{aligned}d^3\sigma_B=K[\sigma_{np}G(k_p')^2+\sigma_{pp}G(k_n')^2\\ +(\Sigma_t+\tfrac{1}{3}\Sigma_s-\sigma_{np}-\sigma_{pp})G(k_p')G(k_n')]. \quad (2.4)\end{aligned}$$

Here, Σ_t and Σ_s are the triplet and singlet parameters given in Eqs. (1.2) and (1.5). The first term in (2.4) is the contribution to the cross section from events in which the observed neutron is actually the recoil particle from a proton-neutron collision. The second term is the contribution in which the incident proton interacted with the target proton and the observed neutron is actually a high-energy spectator particle. This term is very small compared to the first term for all but the very low energy neutrons. The third term is the interference between these two events.

If we neglect altogether the last two terms in Eq. (2.4), we get the spectator-model cross section given in Eq. (1.7). Numerically, we find that the "Born approximation" differs much less from the spectator model than do the calculations which include final-state interactions (discussed below), and usually the differences are in the opposite direction. "Born approximation" and spectator model differ most at small proton scattering angles, 20° and 25° lab. Thus, the ambiguity as to which target particle the incident particle scattered from does not cause important changes to the spectator model for proton angles of 20° or greater.

The "Born-approximation" cross section can be greatly improved upon by including the final-state interaction between the outgoing target nucleons. These particles have a strong S-state interaction for small relative momentum k. Following A, we shall include only interactions in the ^3S and ^1S states. Using singlet and triplet projection operators, the scattering matrix $\mathfrak{M}$ can be written,

$$\begin{aligned}\mathfrak{M}=\Lambda_t(M_{np}+M_{pp})F_t(q,k)\Lambda_t+\Lambda_s(M_{np}+M_{pp})F_s(q,k)\Lambda_t\\ +\{M_{np}[G(k_p')-F_0(q,k)]\\ +M_{pp}[G(k_n')-F_0(q,k)]\}\Lambda_t, \quad (2.5)\end{aligned}$$

where F_t, F_s, and F_0 are the form factors

$$F=\int(\phi_k^{(-)})^*\exp(i\tfrac{1}{2}\mathbf{q}\cdot\mathbf{r})\phi_0 d\mathbf{r}, \quad (2.6)$$

with ${}^t\chi_k^{(-)}$, ${}^s\chi_k^{(-)}$, and $(2\pi)^{-3/2}(\sin kr)/kr$ substituted for $\phi_k^{(-)}$, respectively. Here, ϕ_0 is the deuteron wave function and ${}^t\chi_k^{(-)}$ and ${}^s\chi_k^{(-)}$ are the ^3S and ^1S incoming neutron-proton scattering states of relative momentum k. The first two terms in Eq. (2.5) are the scattering matrices for inelastic events in which the target nucleons are left in a final ^3S and ^1S state, respectively. The last term is the scattering matrix for events which leave the target nucleons in a higher angular momentum state. In this last term, the free S-state form factor F_0 is subtracted from $G(k_n')$ and $G(k_p')$, since the S-state contribution is already included in the first two terms.

Using the form of $\mathfrak{M}$ given in (2.5), we get for the polarized cross section

$$\begin{aligned}&d^3\sigma_{\mathrm{QF}}S_{\mathrm{QF}}(ij)\\ &=\tfrac{1}{6}K\,\mathrm{Tr}[\mathfrak{M}^\dagger\sigma_i\mathfrak{M}\sigma_j]=K\{\tfrac{1}{3}A\Sigma_sS_s(ij)+B\Sigma_tS_t(ij)\\ &\quad+C\sigma_{np}S_{np}(ij)+D\sigma_{pp}S_{pp}(ij)\\ &\quad\quad+E\alpha S_\alpha(ij)+M\beta S_\beta(ij)+N\gamma S_\gamma(ij)\}, \quad (2.7)\end{aligned}$$

where

$$A=|F_s|^2+F_pF_n+(F_p+F_n)\,\mathrm{Re}F_s, \quad (2.8a)$$

$$B=|F_t|^2+F_pF_n+(F_p+F_n)\,\mathrm{Re}F_t, \quad (2.8b)$$

$$C=F_p^2-F_pF_n-(F_n-F_p)(\tfrac{3}{4}\,\mathrm{Re}F_t+\tfrac{1}{4}\,\mathrm{Re}F_s), \quad (2.8c)$$

$$D=F_n^2-F_pF_n+(F_n-F_p)(\tfrac{3}{4}\,\mathrm{Re}F_t+\tfrac{1}{4}\,\mathrm{Re}F_s), \quad (2.8d)$$

$$E=\tfrac{1}{4}(F_n-F_p)(\mathrm{Re}F_t-\mathrm{Re}F_s), \quad (2.8e)$$

$$M=(F_n-F_p)\mathrm{Im}F_s, \quad (2.8f)$$

$$N=(F_n-F_p)\mathrm{Im}F_t. \quad (2.8g)$$

Here,

$$F_n=G(k_n')-F_0, \quad (2.9)$$

$$F_p=G(k_p')-F_0. \quad (2.10)$$

The parameters $\Sigma_t S_t(ij)$ and $\Sigma_s S_s(ij)$ are given by Eqs. (1.2) and (1.6) and are related to elastic and slightly inelastic proton-deuteron scattering; $\sigma_{np}S_{np}$ and $\sigma_{pp}S_{pp}$ are the free neutron-proton and proton-proton polarized cross sections. The other parameters are given by

$$\alpha S_\alpha(ij)=\tfrac{1}{6}\,\mathrm{Tr}\{M_{pp}{}^\dagger\sigma_i(\Lambda_t-3\Lambda_s)M_{pp}\sigma_j\Lambda_t - M_{np}{}^\dagger\sigma_i(\Lambda_t-3\Lambda_s)M_{np}\sigma_j\Lambda_t\}\,, \quad (2.11)$$

$$\beta S_\beta(ij)=\frac{1}{6i}\,\mathrm{Tr}\{M_{np}{}^\dagger\sigma_i\Lambda_s M_{pp}\sigma_j\Lambda_t - M_{pp}{}^\dagger\sigma_i\Lambda_s M_{np}\sigma_j\Lambda_t\}\,, \quad (2.12)$$

$$\gamma S_\gamma(ij)=\frac{1}{6i}\,\mathrm{Tr}\{M_{np}{}^\dagger\sigma_i\Lambda_t M_{pp}\sigma_j\Lambda_t - M_{pp}{}^\dagger\sigma_i\Lambda_t M_{np}\sigma_j\Lambda_t\}. \quad (2.13)$$

These quantities have no simple interpretation in terms of other scattering experiments. However, they can be calculated if M_{np} and M_{pp} are known from some phase-shift analysis.

At the peak for quasifree scattering from the target neutron, the coefficient C in Eq. (2.7) dominates all the others so that measurements of quasifree scattering determine essentially free n-p scattering parameters. However, a quasifree polarization parameter is no longer exactly equal to the corresponding free neutron-proton parameter. Also, the differential cross section given by (2.7) is not the same as that given by (1.7). In Fig. 1 we have plotted these two cross sections as a function of neutron energy for a 142-MeV incident proton scattered through 35° and an included angle θ_{inc} between proton and neutron directions of 87.5°. The SMA cross section has a single peak at $E_n=48.5$ MeV and falls rapidly to zero on either side of this (with a full width at half-maximum of 14 MeV). The final-state approximation, Eq. (2.7), shows three peaks. The largest peak is the quasifree peak and occurs at the same energy as in the SMA. However, this peak is about 10% smaller than the corresponding SMA peak. The second peak at $E_n=13$ MeV is the slightly inelastic peak which occurs when the relative energy in the center-of-mass of the target neutron-proton system is very small. This peak increases as θ_{inc} increases (for fixed proton scattering angle), reaching a maximum when θ_{inc} equals the angle between the scattered proton and an elastically scattered deuteron. On the other hand, the quasifree peak is a maximum at $\theta_{\mathrm{inc}}=87.5°$, the angle between the scattered proton and an elastically scattered neutron, and decreases for greater values of θ_{inc}. Thus, for $\theta_{\mathrm{inc}}=95°$, the ratio of the slightly inelastic to quasifree peak is 0.38, compared to the ratio of 0.05 at $\theta_{\mathrm{inc}}=87.5°$. The very small peak at $E_n=0.36$ MeV corresponds to the observed neutron being the spectator particle for a proton-proton scattering event. The peak is small because the phase space available to such low-energy neutrons is so small. This peak would also occur in the "Born-approximation" cross section, Eq. (2.4), but the slightly inelastic peak is entirely a consequence of including the final S-state interaction between the target nucleons.

In the next Section we shall use Eq. (2.7) to calculate corrections to the quasifree measurements of polarization and triple scattering parameters.

III. EVALUATION AND COMPARISON

The coefficients A, B, C, D, E, M, and N are functions of the incident proton energy, the proton scattering angle θ_p, the neutron scattering angle θ_n, and the neutron energy E_n. These coefficients have been evaluated as a function of E_n for an incident proton energy of 142 MeV and for a variety of proton and neutron scattering angles. These were calculated using the same wave functions as A. That is, F_t and F_s were calculated using a deuteron wave function and ^3S and ^1S scattering wave functions obtained from square well potentials. F_p and F_n were calculated using a Hulthén deuteron wave function. The use of a different deuteron wave function for the different terms in Eqs. (2.8) was done for convenience only. The results are not expected to be sensitive to the details of the deuteron wave function.

The proton-proton quasifree scattering cross section $d^3\sigma/d\Omega_{p1}d\Omega_{p2}dE_{p2}$, in which the incident proton ($p1$) scatters from the target proton ($p2$) and both are detected, is given by Eq. (2.7) also. The coefficients, however, are now given by Eqs. (2.8) with F_p and F_n interchanged everywhere. This means that A and B are unaffected, C and D reverse roles, and E, M, and N change sign. In the following we shall be discussing both p-n and p-p quasifree scattering and it is to be understood that the latter is calculated from Eq. (2.7) with just the above mentioned change in coefficients. For convenience, we shall always refer to the recoil particle as a neutron and continue to use the notation θ_n and E_n for its scattering angle and energy even when referring to quasifree p-p events.

A. Comparison with Everett's Calculation

Everett[11] has calculated corrections to the spectator-model cross section, Eqs. (1.7) and (1.8), by considering the double-scattering terms in a multiple-scattering expansion of the proton-deuteron T operator in terms of the two-body t operators. There are four double-scattering terms, two of which correspond to the successive scattering of the incident particle by the two target nucleons (true double scattering) and two of which correspond to a single scattering of the incident particle by one of the target nucleons followed by the scattering of the target nucleons from each other (final-state interaction). By making a number of suitable approximations, Everett was able to calculate the contribution of these terms. Thus, his results include the effect of true double scattering, whereas in our calculations they are entirely neglected. However, Everett estimated the contribution of the triple-scattering terms to be about 75% of the double-scattering contribution and of the opposite sign. Therefore, it is possible that his calculation overestimates the effect of multiple scattering.

For small proton-scattering angles, Everett found that the major correction to the spectator-model cross section comes from the final-state interaction, which he treats in the same manner as the true double-scattering terms. Since our calculations treat the final S-state interaction exactly, they should agree with Everett's calculations at small angles.

Everett calculated the quasifree p-n cross section, polarization, and the triple-scattering parameters R and A for a proton energy of 145 MeV and proton scattering angles of 20° and 45°. Table I gives the values he obtained for the ratio $d^3\sigma_{\rm QF}/d^3\sigma_{\rm SM}$ of the theoretical quasifree cross section to the value given by the spectator model, and for the quantities $\Delta S_{pn}=S_{np}(\text{free})-S_{pn}(\text{quasifree})$, where $S_{np}(\text{free})$ is one of the free n-p scattering parameters (P, A, or R) and $S_{pn}(\text{quasifree})$ is the value calculated from theory at the quasifree p-n peak. Table I also gives the similar quantities calculated from Eq. (2.7).

Everett used the Gammel-Thaler phase shifts[13] to calculate the nucleon-nucleon scattering matrices, interpolating between the published values at 90 and 156 MeV to obtain values at 145 MeV. In order to make the comparison in Table I meaningful, we have used the same procedure to calculate the quantities $S(ij)$ in Eq. (2.7). However, it is possible that we have used a different interpolation method, since we get different values for some of the free parameters than he does. This is shown in Table I where we have also listed the free n-p parameters P, R, and A as given in Everett's article and as we have calculated from the "same" phase shifts. These differences, unfortunately, preclude a quantitative comparison of the two theories. However, from Table I, it is seen that at 20°, where the final-state interaction is dominant, the two theories give very

TABLE I. Comparison of the corrections to the quasifree p-n scattering parameters calculated by Everett (Ref. 11) with the corrections calculated using Eq. (2.7). The values of the free n-p parameters are also given. (The differences in the free parameters may be due to the different way in which the Gammel-Thaler phase shifts were interpolated to 145 MeV.)

	20°		45°	
θ_p (lab)	Everett	Eq. (2.7)	Everett	Eq. (2.7)
$d^3\sigma_{\rm QF}/d^3\sigma_{\rm SM}$	0.7	0.69	0.87	0.94
P	0.52	0.60	−0.05	+0.05
ΔP	−0.05	+0.02	+0.02	−0.01
R	0.29	0.32	−0.02	−0.24
ΔR	0.20	0.12	0.01	0.00
A	+0.05	−0.05	0.8	0.85
ΔA	−0.17	−0.14	−0.03	0.00

[13] J. L. Gammel and R. M. Thaler, Phys. Rev. **107**, 291 and 1337 (1957).

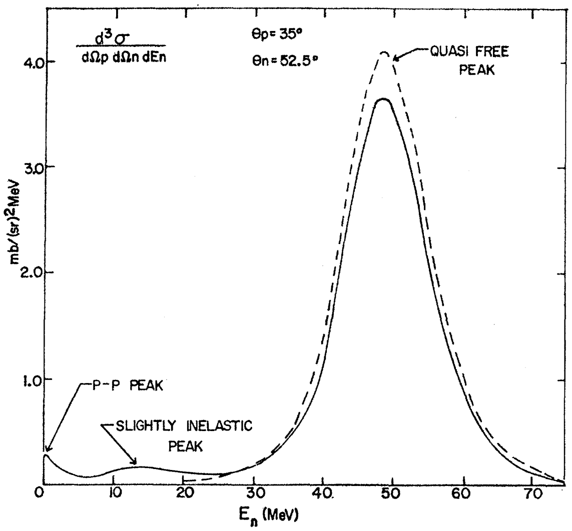

FIG. 1. Proton-deuteron inelastic cross section as a function of recoil neutron energy, as calculated with the spectator model (dashed curve) and with Eq. (2.7), which includes final-state interactions (solid curve). See text for description of the various peaks.

similar results. Thus we get about the same value for $d^3\sigma_{\rm QF}/d^3\sigma_{\rm SM}$ and ΔA as Everett. Our value of ΔR is qualitatively the same, that is, it is large and has the same sign. Our value for ΔP has the opposite sign but this difference is not significant in view of the different values we get for P itself. At 45° our values for the free parameter differ more than the corrections, so it is hard to draw any conclusions, except that both theories agree in giving small corrections at this angle. Our value for $d^3\sigma_{\rm QF}/d^3\sigma_{\rm SM}$ is significantly larger than Everett's and is probably a real effect due to the neglecting of true multiple scattering.

B. Comparison with Quasifree *p-p* Data

One advantage of Eq. (2.7) is that it relates the quasifree cross section more directly to measureable quantities than does Everett's theory. In using (2.7) the parameters $S_{np}(ij)$, $S_{pp}(ij)$, and $S_t(ij)$ were taken to be the free n-p, p-p, and p-d elastic parameters, respectively, and were obtained from experiments.[14] (Exceptions to this were the D parameters for triplet scattering $S_t(33)$ and n-p scattering $S_{np}(33)$, which were calculated from phase shifts as described below.) The uncertainty in each quantity was taken to be the quoted experimental error. The parameters $S_t(33)$, $S_{np}(33)$, $S_s(ij)$, $S_\alpha(ij)$, $S_\beta(ij)$, and $S_\gamma(ij)$ were calculated from the n-p and p-p scattering matrices given

[14] J. N. Palmieri, A. M. Cormack, N. F. Ramsey, and R. Wilson, Ann. Phys. (N.Y.) **5**, 299 (1958); E. H. Thorndike, J. Lefrançois, and R. Wilson, Phys. Rev. **120**, 1819 (1960); S. Hee and E. H. Thorndike (to be published); C. F. Hwang, T. R. Ophel, E. H. Thorndike, and R. Wilson, Phys. Rev. **119**, 352 (1960); W. N. Hess, Rev. Mod. Phys. **30**, 368 (1958); and Refs. 1, 2, 5, and 6.

by the Yale phase-shift solutions,[15] YLAM (for $T=1$) and the 6 YLAN solutions (for $T=0$). The values used were for the $T=0$ solution YLAN-3M and the $T=1$ solution YLAM; uncertainties in the calculated parameters were inferred from the variations between the various $T=0$ solutions. For a given value of θ_p, θ_n and E_n the value of $S(ij)$ appropriate to the angle θ_p and the incident proton energy was used. This amounts to neglecting a small dependence of the momentum transfer q on θ_n and E_n. (The momentum transfer changes by only a few percent over the relevant ranges of θ_n and E_n.)

Most quasifree experiments measure the integrated cross section

$$d^2\sigma_{\mathrm{QF}}S(ij)=\int W(E_n)\left(\frac{d^3\sigma}{d\Omega_n d\Omega_p dE_n}\right)S(ij)dE_n,$$

where $W(E_n)$ is a weighting function which depends on the efficiency of the particular recoil particle detector. In our calculations, we have integrated the cross section from some $E_n(\min)$ to some $E_n(\max)$ using a constant weighting function. This is a fairly reasonable approximation for both recoil proton and recoil neutron detectors. As can be seen from Fig. 1, the integrated cross section may be sensitive to the value of $E_n(\min)$ used. The uncertainty to the cross section arising from the uncertainty in $E_n(\min)$ was obtained by calculating

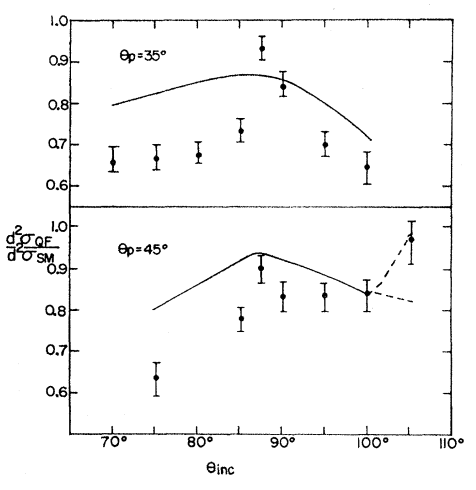

FIG. 2. The ratio $d^2\sigma_{\mathrm{QF}}/d^2\sigma_{\mathrm{SM}}$ of quasifree p-p cross section to spectator model prediction as a function of θ_{inc}, the angle between the two protons, for two proton scattering angles ($\theta_p=35°$ and 45°). The cross sections have been integrated over recoil particle energy. The points are measurements of KWC, Ref. 4. The curves are values for this ratio calculated from Eqs. (2.7) and (1.7). The dashed portion of the curve for $\theta_p=45°$ indicates the range allowed by the uncertainty in the minimum detectable energy of recoil proton in that region.

[15] G. Breit, M. Hull, K. Lassila, and K. Pyatt, Jr., Phys. Rev. **120**, 2227 (1960); M. Hull, K. Lassila, H. Ruppel, F. McDonald, and G. Breit, *ibid.* **122**, 1606 (1961); and private communications. We wish to thank Professor Breit for sending us scattering amplitudes calculated from these phase shifts.

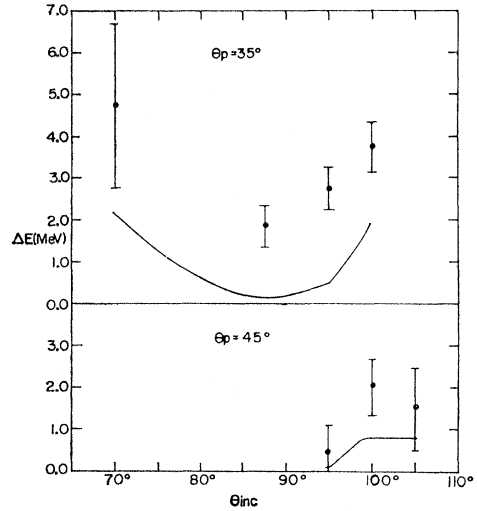

FIG. 3. ΔE (the reduction, in the full width at half-maximum, below the spectator model width, of the quasifree p-p cross section spectra as functions of recoil proton energy) as a function of θ_{inc}, the angle between the two protons, for two proton scattering angles ($\theta_p=35°$ and 45°). The points are measurements of KWC, Ref. 4. The curves are values for ΔE calculated from Eqs. (2.7) and (1.7).

the change in the cross section caused by varying $E_n(\min)$ to approximate extreme possible detector efficiencies. This uncertainty was combined with the uncertainties in the different input parameters $S(ij)$ to obtain the statistical uncertainty in the calculated quantities.

Kuckes, Wilson, and Cooper[4] (hereafter, KWC) have made measurements of the quasifree p-p cross section and polarization. In Fig. 2, the ratio $d^2\sigma_{\mathrm{QF}}/d^2\sigma_{\mathrm{SM}}$ of their cross sections (integrated over the recoil particle energy) to the spectator model prediction is shown as a function of θ_{inc}, the angle between the two detectors, for two proton scattering angles ($\theta_p=35°$ and 45°). The solid curves are the calculated values for this ratio. The dashed portion of the curve for $\theta_p=45°$ indicates the range allowed by the uncertainty in the minimum detectable energy of the recoil proton in this region. Elsewhere, the statistical error in the calculated curve is not greater than ± 0.03. Not shown in Fig. 2 is a measurement at $\theta_p=30°$, $\theta_{\mathrm{inc}}=75°$ of 0.63 ± 0.04, to be compared with the calculated value of 0.80. These calculations consistently underestimate the reduction in the cross section below the spectator model, but do account for at least 50% of the reduction. Everett's calculations,[11] which include the effect of true double scattering, are in better agreement with the KWC values, and if anything, overestimate the reduction.

In comparing the shape (as a function of recoil proton energy) of the unintegrated cross section, KWC found that the spectra were narrower than predicted by the spectator model. The reduction in the full width at half-maximum, below the spectator model width, is shown in Fig. 3. The solid curve is the cal-

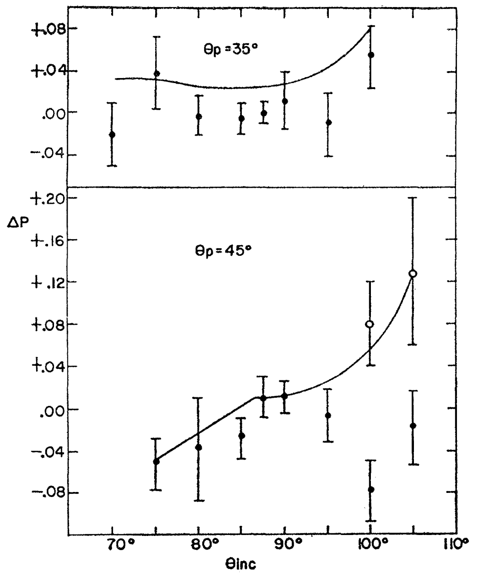

FIG. 4. ΔP (the difference between the free p-p polarization and quasifree p-p polarization) as a function of $\theta_{\rm inc}$, the angle between the two protons, for two proton scattering angles ($\theta_p=35°$ and 45°). The points are measurements of KWC, Ref. 4. The curves are values for ΔP calculated from Eq. (2.7). See text for explanation of the two sets of experimental points for $\theta_p=45°$, large $\theta_{\rm inc}$.

culated value, which again accounts for only a portion of the reduction.

Figure 4 shows the values measured by KWC of ΔP, the difference between the free p-p polarization and quasifree polarization, as a function of $\theta_{\rm inc}$. The solid curve gives the calculated values. Table II shows some additional data. At 35° the calculation is high by at most 0.02. At 45° the agreement is very good. Two sets of data are shown at the two largest included angles, taken from separate experimental runs. The difference between the two sets of data is believed to be due to the subtraction for elastic-scattering contamination. KWC prefer the set that *disagrees* with the calculated curve, feeling that the other set contains some elastic scattering. However, it is quite possible that KWC have subtracted some slightly inelastic scattering in obtaining the set they prefer, and that the points agreeing with the curve are, in fact, correct. Thus, there is evidence that the calculation disagrees by no more than ± 0.02 with any of the p-p polarization data.

In I the quasifree p-p triple-scattering parameters R and A were measured and compared with the measured

TABLE II. Additional values of ΔP_{pp} not shown in Fig. 4.

θ_p	$\theta_{\rm inc}$	ΔP (meas)	ΔP (theory)
30°	87.5°	$+0.016\pm0.020$	$+0.048$
30°	90°	-0.003 ± 0.018	$+0.054$
40°	85°	$+0.023\pm0.020$	...
40°	87.5°	-0.030 ± 0.020	$+0.018$
40°	90°	0.000 ± 0.017	...

values of the free p-p parameters. Their differences [i.e., $\Delta S=S(\text{free})-S(\text{quasifree})$, where S is a scattering parameter] are shown in Table III together with the calculated values for these quantities. These calculated values were obtained by integrating over the relevant range of recoil proton energies, as discussed earlier. In addition, the cross section was averaged over the range of recoil proton angles accepted by the large recoil counters ($\pm 12°$). All that can be concluded from Table III is that there is no evidence of any serious discrepancy within the rather large experimental errors.

C. Corrections to Quasifree *p-n* Data

An important purpose of this work is to obtain corrections which can be used to convert quasifree p-n scattering measurements to free n-p scattering parameters. These corrections have been calculated for the polarization measurements of Kuckes and Wilson[5] and for the R and A measurements of I. Table IV gives the quasifree p-n polarization measurements of Kuckes and Wilson, the calculated corrections, and the inferred free n-p polarization. Similar data for the R and A measurements of I are given in Tables V and VI.

TABLE III. ΔR_{pp} and ΔA_{pp} as given by the measurements of I and by theory.

θ_p (lab)	ΔR (meas)	ΔR (theory)	ΔA (meas)	ΔA (theory)
30°	$+0.09\pm0.07$	$+0.014$	$+0.01\pm0.10$	$+0.011$
35°	$+0.13\pm0.08$	$+0.004$	-0.03 ± 0.10	$+0.035$
40°	$+0.04\pm0.15$	$+0.015$	-0.10 ± 0.14	$+0.018$

Although no quasifree measurements of D have yet been made at this energy, Table VII gives the corrections to D for quasifree p-n scattering, assuming the same experimental conditions as those of R and A in I.

In these calculations, Eq. (2.7) has been integrated over the relevant range of E_n and θ_n, as discussed in Sec. III B. The statistical errors listed in Tables IV, V, and VI are those due to uncertainties in the values of the parameters $S(ij)$ used in Eq. (2.7) and to the uncertainty in $E_n(\text{min})$. (In Tables V and VI, a small error from averaging over recoil particle directions is also included.) In addition to these statistical uncertainties, there is in each case an uncertainty due to the limited validity of the theory. It is very difficult to estimate the validity *a priori* without making a detailed study of the multiple-scattering effects.

However, from the comparison with the quasifree p-p data in Sec. III B we can make some plausible *a posteriori* inferences about the accuracy of the theory. It was found that theory accounts for better than half of the deviation of the quasifree p-p cross section from the spectator model, it is consistent with the values of R and A to within their experimental errors, typically ± 0.10, and it gives the polarization to at least ± 0.02.

TABLE IV. Polarization in quasifree p-n scattering (Ref. 5), the correction to it, and the inferred free n-p scattering polarization parameter. There is a systematic error of $[(0.04)^2+(\frac{1}{4}\Delta p)^2]^{1/2}$ in the inferred free parameter (see text) which dominates the listed statistical error.

θ_p (lab)	P_{pn} (quasifree)	ΔP	P_{np} (free)	θ (c.m.)
20°	0.475±0.039	0.051±0.013	0.526±0.041	41°
25°	0.495±0.017	0.031±0.011	0.526±0.020	51°
30°	0.480±0.016	−0.002±0.010	0.478±0.019	62°
35°	0.425±0.021	−0.033±0.010	0.392±0.023	72°
40°	0.272±0.021	−0.046±0.011	0.226±0.024	82.5°
45°	0.160±0.015	−0.049±0.010	0.111±0.018	92.5°

Furthermore, the corrections it gives to the n-p parameters at 45° (Table I) agree with Everett's values to within ±0.04. It seems reasonable to assume, therefore, that where the theory gives a small correction ($\lesssim 0.04$) this correction itself is good to at least ±0.04.

We believe that the theory should give corrections to the polarization and triple-scattering parameters somewhat better than it does to the cross section, simply because the former, being essentially ratios of cross sections, will not be as sensitive to true multiple-scattering effects. Thus, where the calculated correction is large ($\gtrsim 0.04$) it is expected to account for 75% of the effect and so should be good to $\pm\frac{1}{4}\Delta S$. In conclusion we would assign a validity uncertainty to a calculated correction ΔS of $[(0.04)^2+(\frac{1}{4}\Delta S)^2]^{1/2}$. This is a systematic error, varying smoothly with scattering angle. For the polarization data, Table IV, it is the dominant uncertainty. For R and A, Tables V and VI, the experimental errors dominate.

There have been recent measurements of the free n-p polarization parameter near 140 MeV by Stafford and Whitehead.[16] Their neutron beam had an energy spectrum from 100 to 190 MeV; and hence, it would not be correct to treat their measurements as 140-MeV data without applying corrections for the energy spectrum (with their corresponding errors). Without such corrections, the most one can say is that the two sets of data are in general agreement. Both before and after the corrections have been applied to the quasifree data, differences are typically ±0.05, but reach 0.10 at 62° c.m., where Stafford and Whitehead have a point that looks anomalously high compared to the rest of their points.

TABLE V. R in quasifree p-n scattering (from I), the correction to it, and the inferred free n-p scattering R parameter. There is a systematic error of $[(0.04)^2+(\frac{1}{4}\Delta R)^2]^{1/2}$ in the inferred free parameter (see text) in addition to the listed statistical error.

θ_p	R_{pn} (quasifree)	ΔR	R_{np} (free)	θ (c.m.)
20°	+0.029±0.080	0.140±0.039	+0.169±0.089	42°
25°	−0.006±0.082	0.086±0.025	+0.080±0.086	$52\frac{1}{2}$°
30°	−0.061±0.063	0.038±0.016	−0.023±0.065	63°
35°	−0.160±0.089	0.009±0.011	−0.151±0.090	$73\frac{1}{2}$°
40°	−0.164±0.207	0.018±0.014	−0.146±0.207	$83\frac{1}{2}$°

[16] G. H. Stafford and C. Whitehead, Proc. Phys. Soc. (London) **79**, 430 (1962).

TABLE VI. A in quasifree p-n scattering (from I), the correction to it, and the inferred free n-p scattering A parameter. There is a systematic error of $[(0.04)^2+(\frac{1}{4}\Delta A)^2]^{1/2}$ in the inferred free parameter (see text) in addition to the listed statistical error.

θ_p	A_{pn} (quasifree)	ΔA	A_{np} (free)	θ (c.m.)
20°	0.052±0.072	−0.072±0.028	−0.020±0.077	42°
25°	0.123±0.059	−0.053±0.017	+0.070±0.060	$52\frac{1}{2}$°
30°	0.214±0.076	−0.004±0.021	+0.210±0.079	63°
35°	0.098±0.095	+0.028±0.018	+0.126±0.096	$73\frac{1}{2}$°
40°	0.496±0.216	+0.036±0.016	+0.532±0.216	$83\frac{1}{2}$°

There are no free n-p measurements to compare with the corrected values of R and A from Tables V and VI. However, a comparison can be made with the values calculated from the 6 Yale phase-shift solutions,[15] YLAN, and with a Gammel-Thaler solution.[7] At 42° c.m., the phase-shift values for R range from 0.17 to 0.23; the uncorrected experimental value is 0.03 and the corrected value is 0.17. At other angles, and for the A parameter, both uncorrected and corrected values are bracketed by the various phase-shift values. (The A measurement at 73° c.m. is an exception.) Corrections usually move the points towards solutions YLAN 3 or 3M, which are believed to be the best solutions on other grounds. There is no indication from these comparisons that our estimate of the accuracy of the corrections is in error by more than a factor of 2. (Further discussion of the corrected values of R and A appears in I.)

IV. SINGLET SCATTERING

In the region of slightly inelastic proton-deuteron scattering, where the relative energy in the c.m. of the target nucleons is small, the cross section is dominated by the first term in Eq. (2.7). Measurements of the cross section, the polarization, and the triple scattering parameters of the inelastically scattered proton in this region can, thus, determine the singlet parameters Σ_s and $S_s(ij)$ given in Eqs. (1.5) and (1.6).

The nucleon-nucleon scattering matrix, in an isotopic spin state T ($T=0$ or 1), can be written in the form[7,8]

$$M_T=A_T+B_T(\boldsymbol{\sigma}_1\cdot\mathbf{n})(\boldsymbol{\sigma}_2\cdot\mathbf{n})+C_T(\boldsymbol{\sigma}_1\cdot\mathbf{n}+\boldsymbol{\sigma}_2\cdot\mathbf{n})+E_T(\boldsymbol{\sigma}_1\cdot\mathbf{q})(\boldsymbol{\sigma}_2\cdot\mathbf{q})+F_T(\boldsymbol{\sigma}_1\cdot\mathbf{p})(\boldsymbol{\sigma}_2\cdot\mathbf{p}),$$

TABLE VII. Corrections to quasifree p-n scattering measurements of the D parameter.

θ_p	ΔD
20°	0.10
25°	0.08
30°	0.08
35°	0.06
40°	0.02

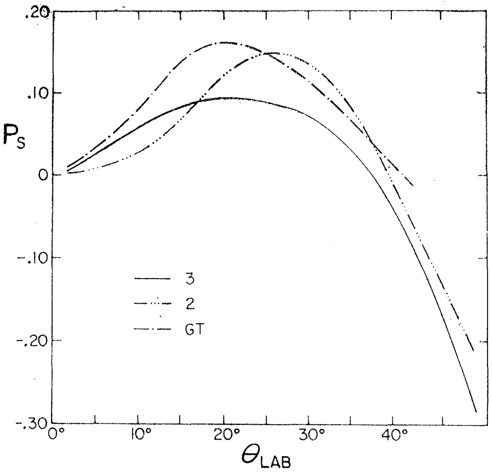

FIG. 5. P_s, the polarization in singlet p-d scattering, versus proton scattering angle θ_{lab}, as predicted using the nucleon-nucleon phase-shift solutions of Breit and collaborators, Ref. 15, at 140 MeV, and of Gammel and Thaler, Ref. 7, at 156 MeV.

TABLE VIII. Singlet scattering parameters in impulse approximation.

$$\Sigma_s = |B|^2+|C|^2+|E|^2+|F|^2$$
$$\Sigma_s P = 2\,\mathrm{Re}CB^*$$
$$\Sigma_s D = |C|^2+|B|^2-|E|^2-|F|^2$$
$$\Sigma_s X = |C|^2-|B|^2$$
$$\Sigma_s Y = |E|^2-|F|^2$$
$$\Sigma_s Z = 2\,\mathrm{Im}CB^*$$
$$R = X\cos\theta_{\mathrm{lab}}+Y\cos(\theta_{\mathrm{lab}}-2\alpha)+Z\sin\theta_{\mathrm{lab}}$$
$$A = -X\sin\theta_{\mathrm{lab}}-Y\sin(\theta_{\mathrm{lab}}-2\alpha)+Z\cos\theta_{\mathrm{lab}}$$
$$R' = X\sin\theta_{\mathrm{lab}}-Y\sin(\theta_{\mathrm{lab}}-2\alpha)-Z\cos\theta_{\mathrm{lab}}$$
$$A' = X\cos\theta_{\mathrm{lab}}-Y\cos(\theta_{\mathrm{lab}}-2\alpha)+Z\sin\theta_{\mathrm{lab}}$$
$$\alpha = \theta_{\mathrm{lab}}(\text{proton})+\phi_{\mathrm{lab}}(\text{momentum transfer})-90°$$
$$B = B_{np}-B_{pp} = \tfrac{1}{2}(B_0-B_1),\ \text{etc.}$$

where $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are the spin operators for the two nucleons and $\mathbf{n}$, $\mathbf{p}$, and $\mathbf{q}$ are the unit vectors defined in Sec. I. Table VIII gives the expressions for the various singlet parameters in terms of these amplitudes. These parameters are bilinear combinations of the quantities $B=\frac{1}{2}(B_0-B_1)=B_{np}-B_{pp}$, etc., and so are expected to be particularly sensitive to the elusive $T=0$ amplitudes.

Figures 5 through 9 show the values of the singlet parameters P_s, R_s, A_s, D_s, and A_s', respectively, predicted by the nucleon-nucleon phase-shift solutions of Breit and his collaborators[15] at 140 MeV. The p-p amplitudes were all calculated from the YLAM phase-shift solution and the n-p amplitudes were calculated from the various solutions YLAN 0, 1, 2, 2M, 3, and 3M, as indicated. The predictions of the Gammel-Thaler n-p and p-p amplitudes[7] at 156 MeV for P_s and R_s are also given. It is seen that for D_s, R_s, and A_s', the predictions of the various YLAN solutions differ considerably, especially in the small-angle region ($\leq 10°$). It thus appears that a measurement at 5° lab of D_s or R_s to an accuracy of ± 0.1 or of A_s' to ± 0.2 could appreciably restrict the n-p phase-shift solutions. Fortunately, it is just in this small-angle region that the singlet contribution to the slightly inelastic scattering is most dominant and that the theory of A is most valid. Furthermore, this is the region in which free n-p parameters are most difficult to measure.

Two methods of measuring the singlet parameters present themselves. In the first method one measures the polarized cross section $(d^2\sigma/d\Omega_p dE_p)S(ij)$, of the protons inelastically scattered from deuterium near threshold, without detecting a recoil particle. Stairs *et al.*[3] have measured the unpolarized cross section as a function of the proton energy at laboratory angles of 5°, 10°, 15°, and 20°. In A these data were analyzed to

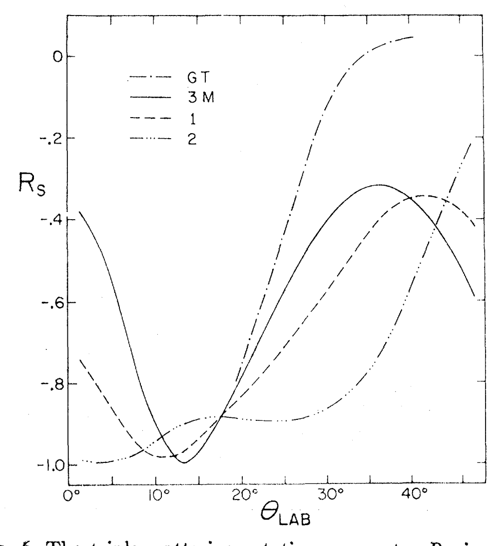

FIG. 6. The triple-scattering rotation parameter R_s, in singlet p-d scattering, versus proton scattering angle θ_{lab}, as predicted using the nucleon-nucleon phase-shift solutions of Breit and collaborators, Ref. 15, at 140 MeV, and Gammel and Thaler, Ref. 7, at 156 MeV.

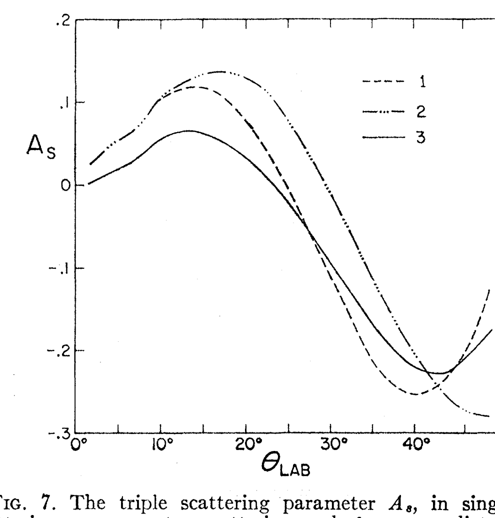

FIG. 7. The triple scattering parameter A_s, in singlet p-d scattering, versus proton scattering angle θ_{lab}, as predicted using the nucleon-nucleon phase-shift solutions of Breit and collaborators, Ref. 15, at 140 MeV.

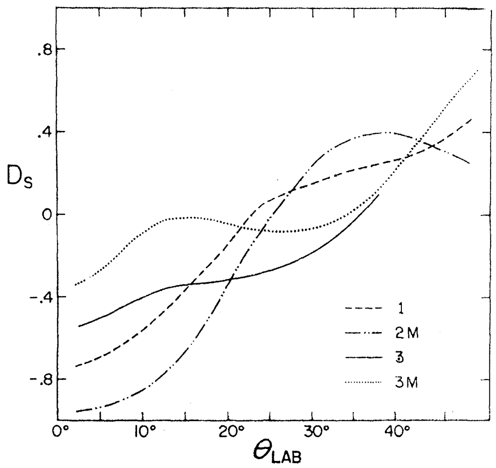

FIG. 8. The triple scattering depolarization parameter D_s, in singlet *p-d* scattering, versus proton scattering angle θ_{lab}, as predicted using the nucleon-nucleon phase-shift solutions of Breit and collaborators, Ref. 15, at 140 MeV.

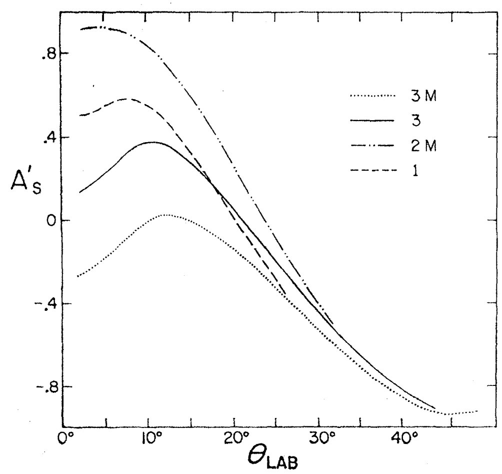

FIG. 9. The triple-scattering parameter A_s', in singlet *p-d* scattering, versus proton scattering angle θ_{lab}, as predicted using the nucleon-nucleon phase-shift solutions of Breit and collaborators, Ref. 15, at 140 MeV.

determine $\sum_s$ at these angles. The results were found to be about 40% larger than the predictions of solution YLAM+YLAN 3M. However, in order to separate the inelastic scattering from the elastic scattering, very good energy resolution was required ($\pm 0.4\%$). It would be difficult to attain this resolution in a triple-scattering experiment and, thus, D, R, and A' could perhaps be more readily measured by the second method.

This second method is based on Eq. (2.7) and involves detecting both the scattered proton and the recoil neutron, thus assuring a separation of inelastic from elastic events. At small scattering angles and for *low-recoil neutron energies* the first term in Eq. (2.7) is dominant so that such events will essentially determine the singlet parameters and corrections can be made for the contribution of the other terms. Thus at an incident energy of 140 MeV and for $\theta_p = 5°$, the recoil neutron spectrum peaks at an energy of 0.3 MeV. If the recoil neutrons with energies between 0.1 and 1.0 MeV were detected, the major portion of the singlet scattering would be included and the other six terms in (2.7) would contribute only 3% of the scattering events. Counting rates in such an experiment would be much smaller than in the usual quasifree experiments[4–6] because of the smaller cross section and the smaller recoil counter solid angle dictated by the sharper correlation between the directions of the proton and neutron.

We conclude that an experiment to measure the singlet triple-scattering parameters, while difficult, does appear to be feasible. Moreover, a measurement of these singlet parameters should be at least as important in *n-p* phase shift analyses as the free *n-p* triple-scattering parameters.

ACKNOWLEDGMENTS

We gratefully acknowledge the support and encouragement of Professor Richard Wilson. We wish to thank Professor Breit for supplying us with nucleon-nucleon amplitudes. Discussions with Dr. A. Everett proved useful. Some of the calculations were performed by P. Koehler and D. Owen, and Miss A. Bailey, making use of the Cambridge Electron Accelerator G15 computer and University of Rochester 7070 computer. This work was done in part at the Computation Center at Massachusetts Institute of Technology, and we gratefully acknowledge the cooperation of the staff there.